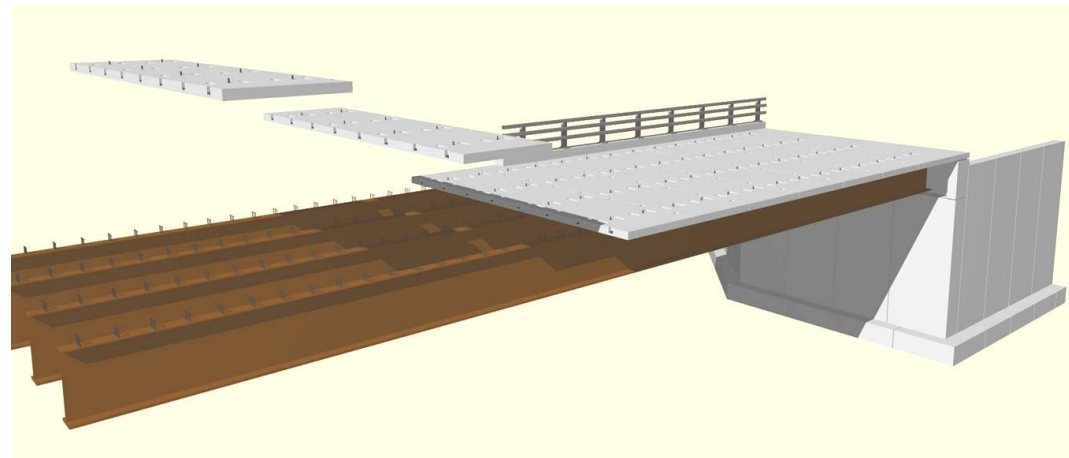
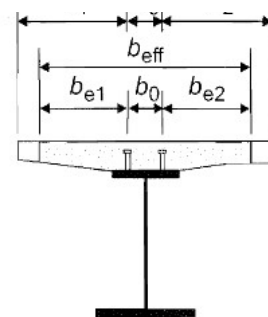
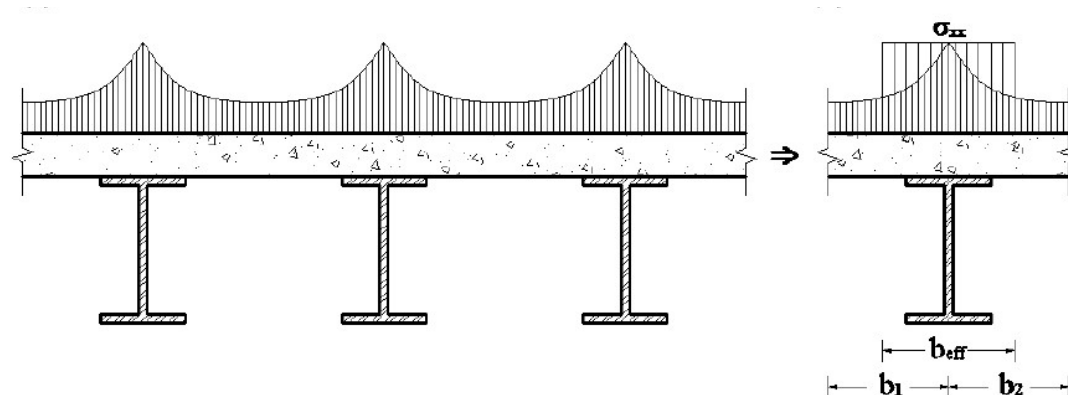


# Um elemento finito fisicamente não-linear para vigas mistas aço-betão



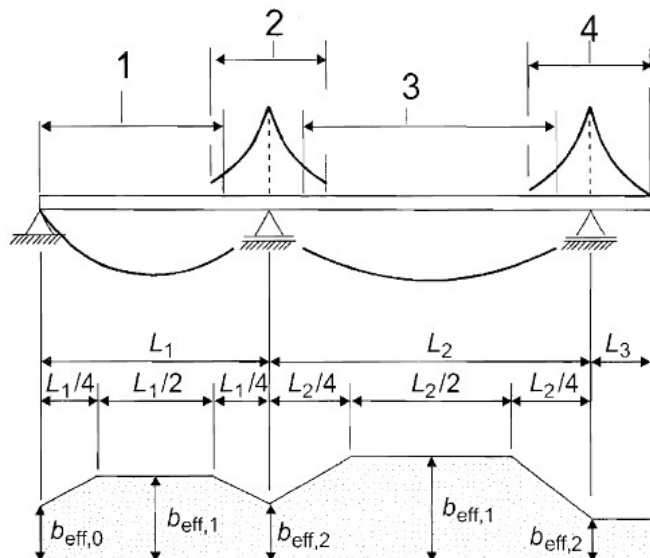
# Motivação

- Shear Lag;
- Fendilhação.



Key:

- 1  $L_c = 0,85 L_1$  for  $b_{eff,1}$
- 2  $L_c = 0,25(L_1 + L_2)$  for  $b_{eff,2}$
- 3  $L_c = 0,70 L_2$  for  $b_{eff,1}$
- 4  $L_c = 2 L_3$  for  $b_{eff,2}$

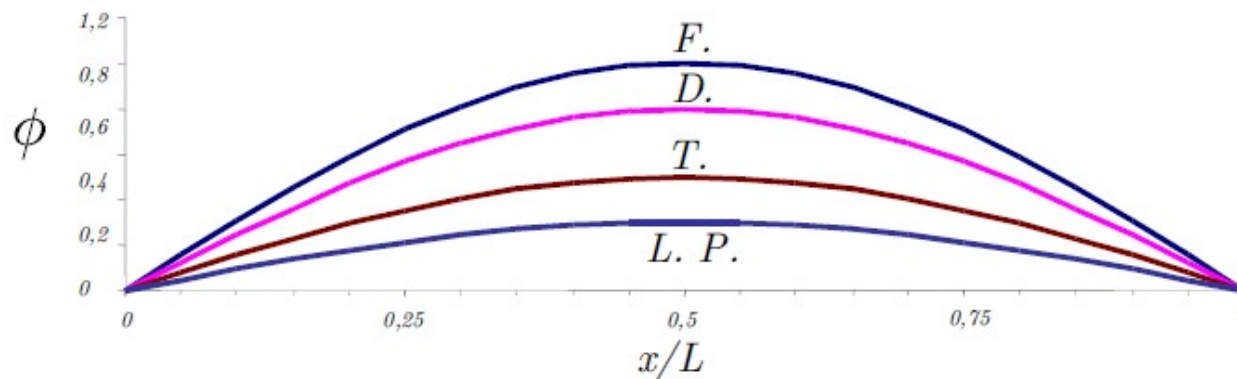
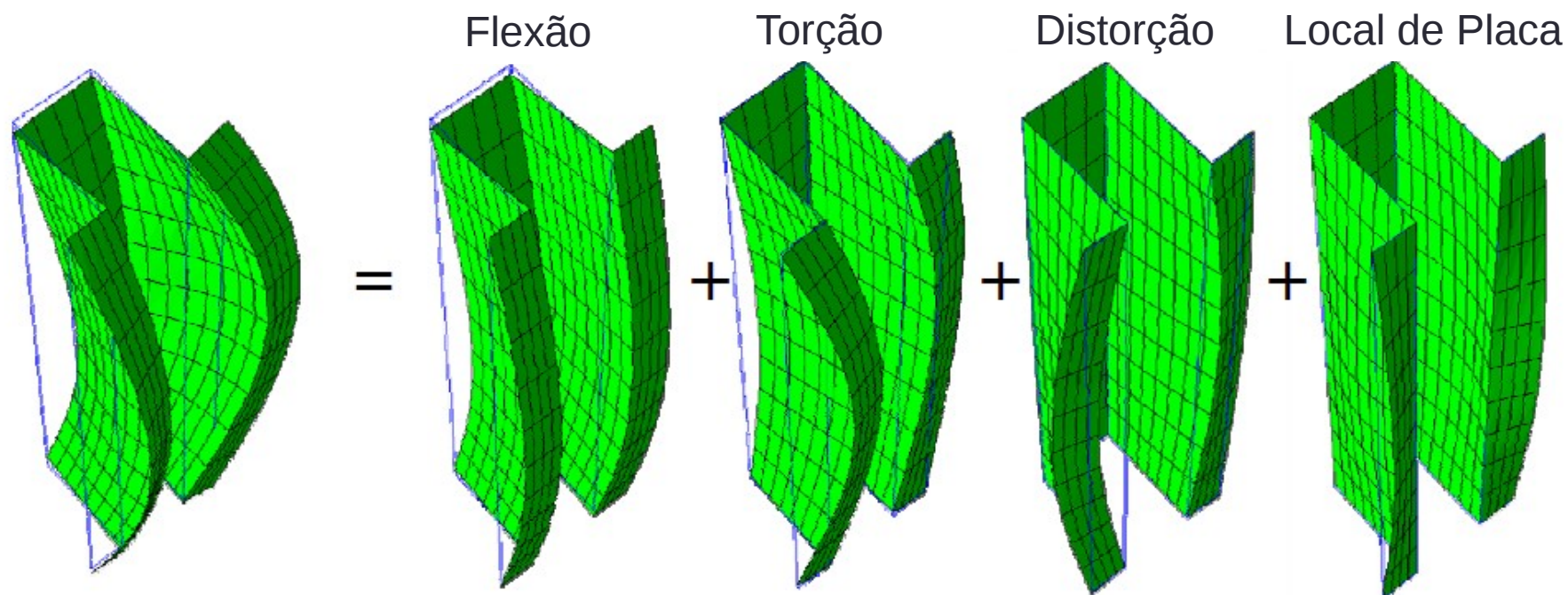


# Objetivos

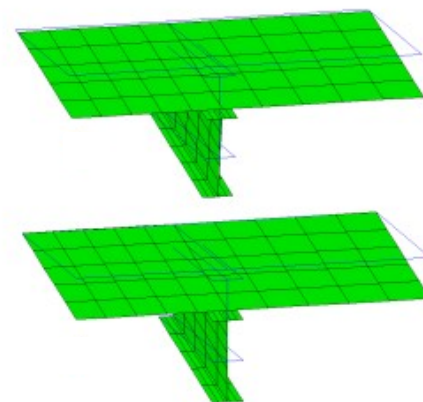
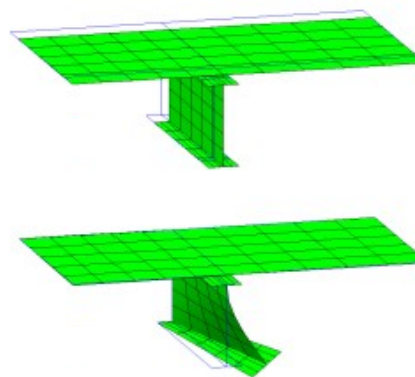
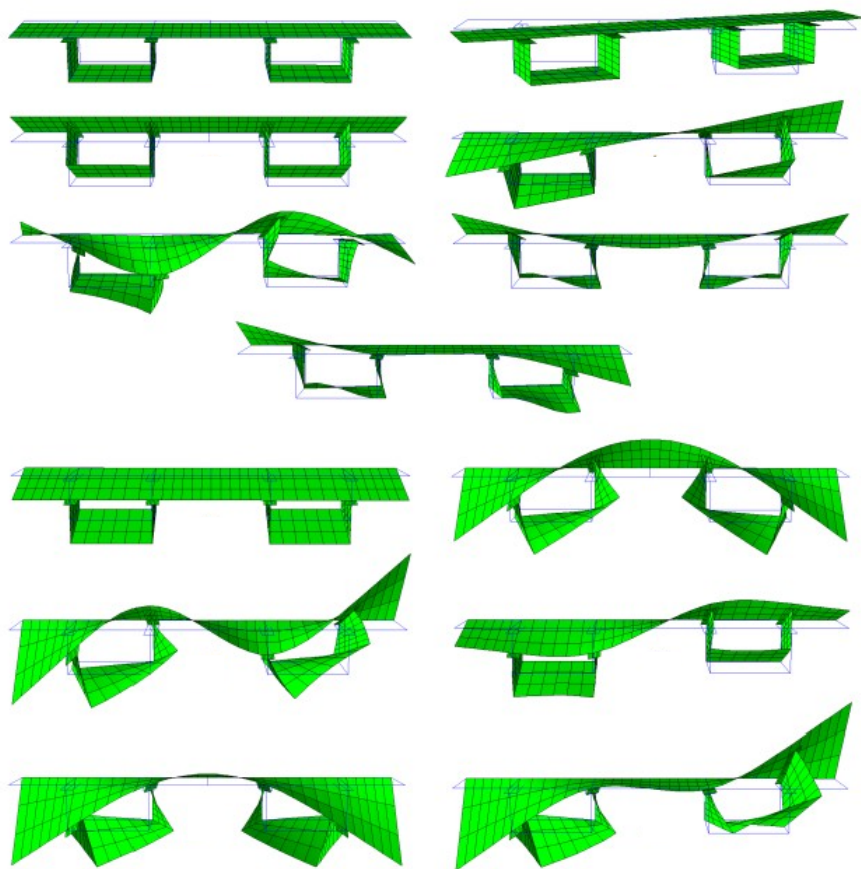
---

- Elemento finito não-linear baseado na GBT;
- Programa em MATLAB:
  - análises não-lineares com o elemento finito proposto;
  - visualizador de resultados;
  - adaptável para futura incorporação de novas funcionalidades;

# Teoria Generalizada de Vigas



# Modos de deformação



# Formulação em elementos finitos

- Deslocamentos

$$\mathbf{U}(x, y, z) = \begin{bmatrix} u - z w_{,x} \\ v - z w_{,y} \\ w \end{bmatrix}$$

- Matriz de Rigidez

$$\mathbf{K}_t = \int_V \begin{bmatrix} \Psi \\ \Psi_{,x} \\ \Psi_{,xx} \end{bmatrix}^t \Xi_{\varepsilon}^t \mathbf{C}_t \Xi_{\varepsilon} \begin{bmatrix} \Psi \\ \Psi_{,x} \\ \Psi_{,xx} \end{bmatrix} dV$$

- Vetor das forças internas

$$\mathbf{Q} = \int_V \begin{bmatrix} \Psi \\ \Psi_{,x} \\ \Psi_{,xx} \end{bmatrix}^t \Xi_{\varepsilon}^t \boldsymbol{\sigma} dV$$

- Deformações

$$\begin{aligned} \varepsilon_{xx} &= U_{x,x}, & \gamma_{xy} &= \gamma_{yx} = U_{x,y} + U_{y,x}, \\ \varepsilon_{yy} &= U_{y,y}, & \gamma_{xz} &= \gamma_{zx} = U_{x,z} + U_{z,x}, \\ \varepsilon_{zz} &= U_{z,z}, & \gamma_{yz} &= \gamma_{zy} = U_{z,y} + U_{y,z}. \end{aligned}$$

- Vetor das forças exteriores

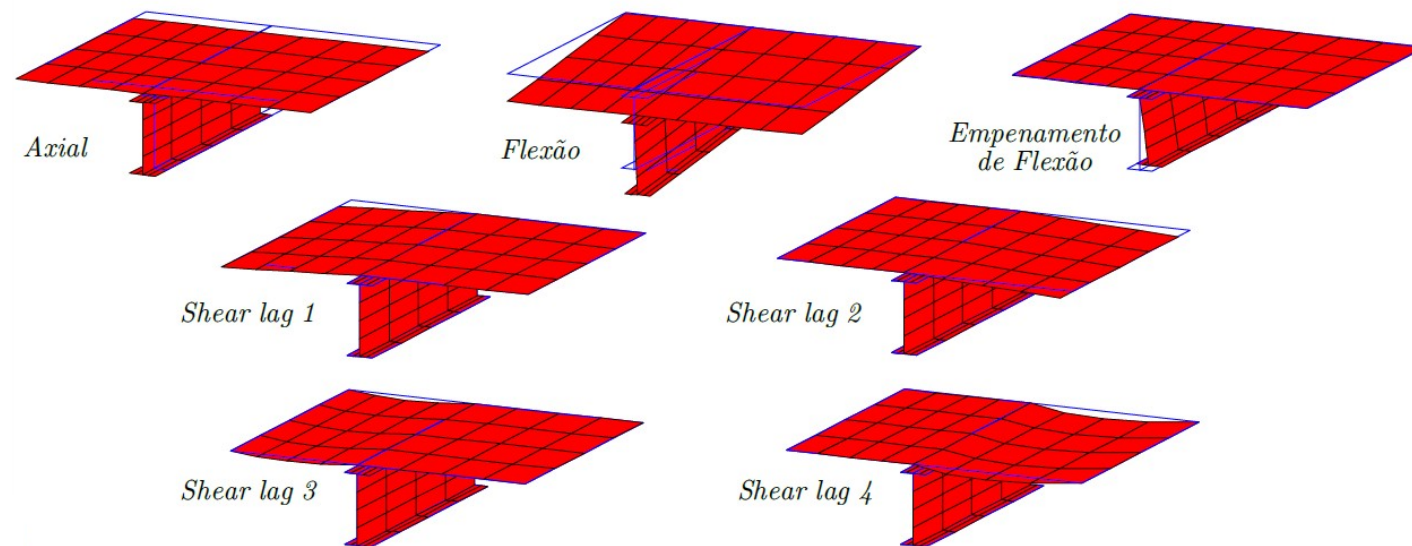
$$\mathbf{F} = \int_{\Omega} \begin{bmatrix} \Psi \\ \Psi_{,x} \end{bmatrix}^t (\Xi_U^M)^t \bar{\mathbf{f}} d\Omega$$

- Vetor das forças desequilibradas

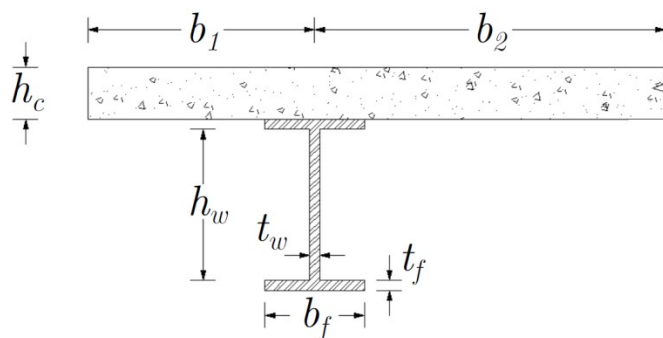
$$\mathbf{g} = \int_V \begin{bmatrix} \Psi \\ \Psi_{,x} \\ \Psi_{,xx} \end{bmatrix}^t \Xi_{\varepsilon}^t \boldsymbol{\sigma} dV - \int_{\Omega} \begin{bmatrix} \Psi \\ \Psi_{,x} \end{bmatrix}^t (\Xi_U^M)^t \bar{\mathbf{f}} d\Omega$$

# Hipóteses simplificativas

## Secção Discretizada e Modos de Deformação

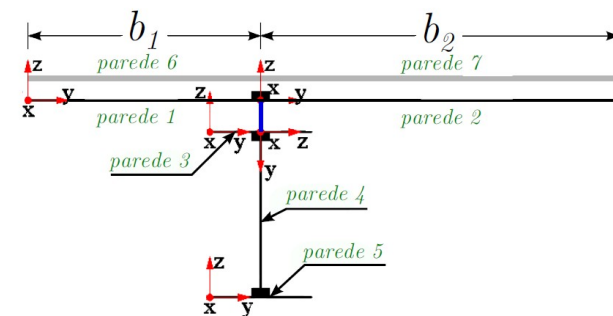


- $\epsilon_{yy}^M = \epsilon_{yy}^B = 0$
- $\gamma_{xy}^M = \gamma_{xy}^B = 0$
- $\sigma_{yy} = 0$



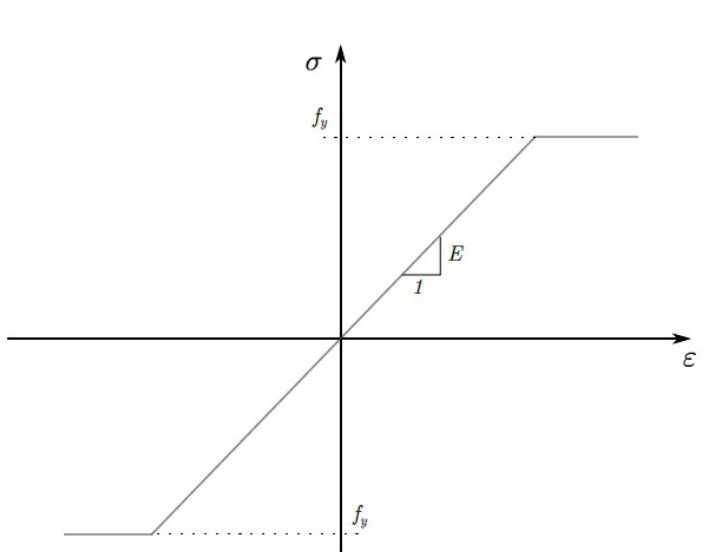
$$h_c/2 + t_f/2$$

$$h_w - t_f$$

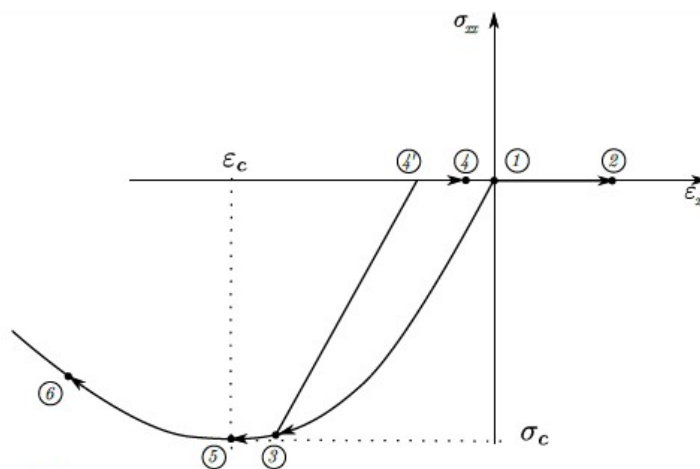


# Leis constitutivas

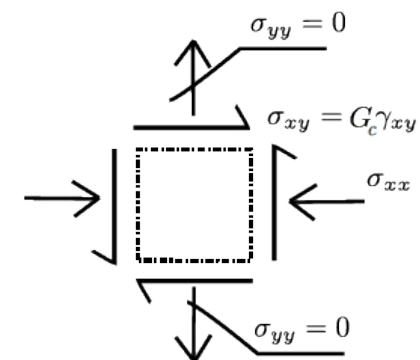
## Aço



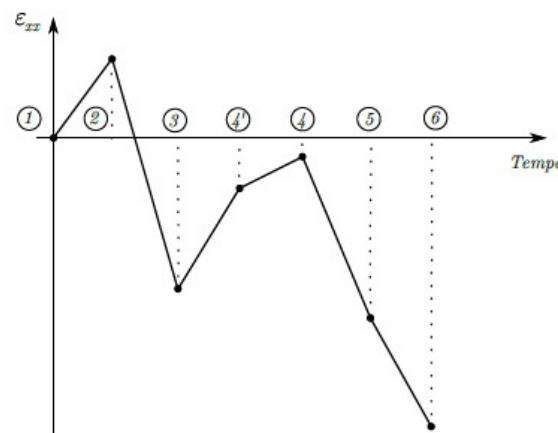
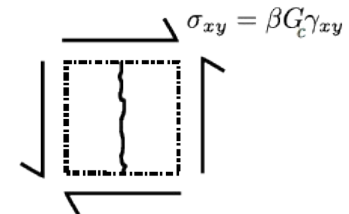
## Betão



*Estado não fendilhado*



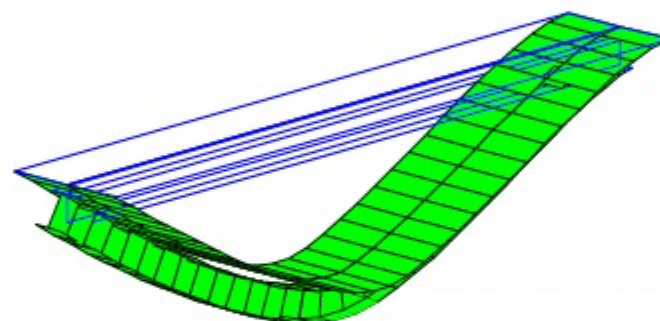
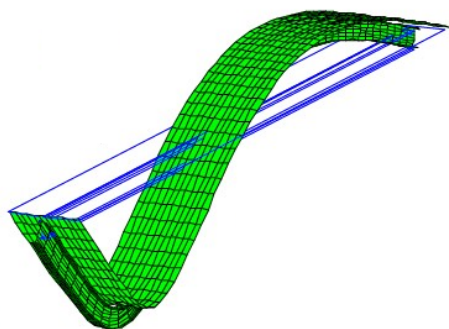
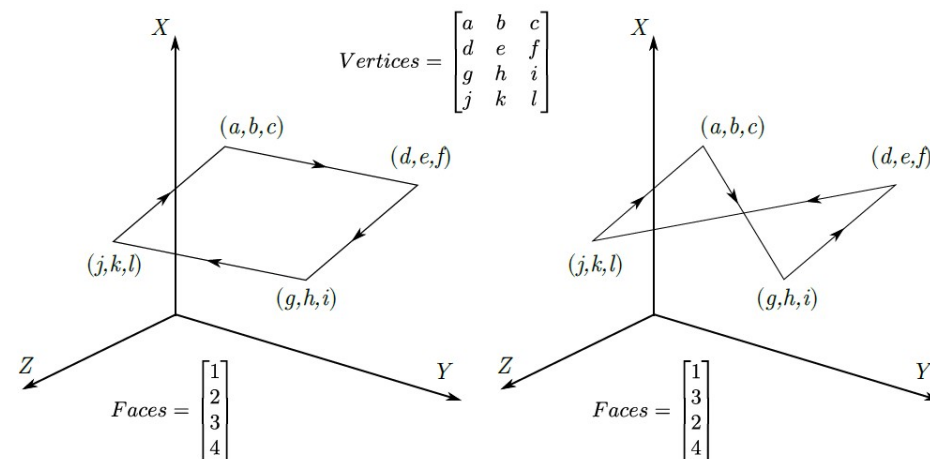
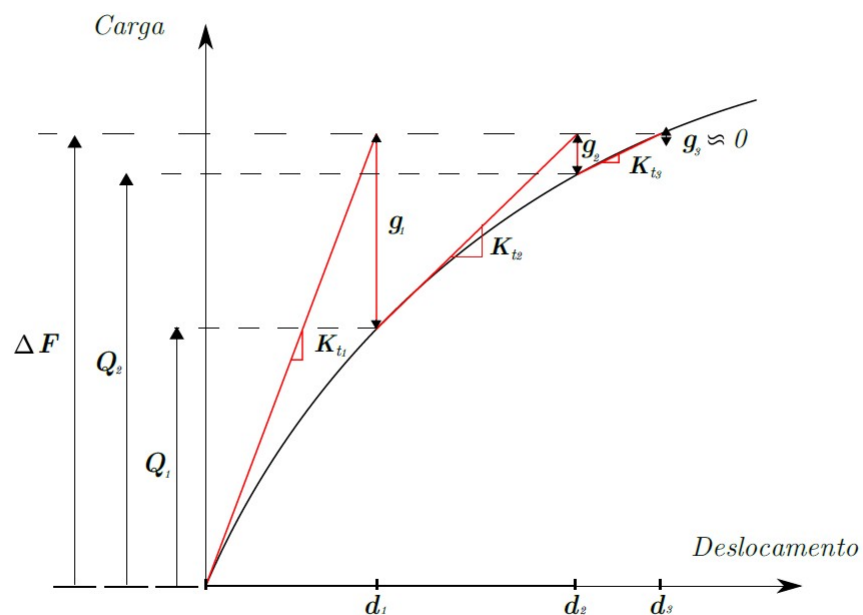
*Estado fendilhado*



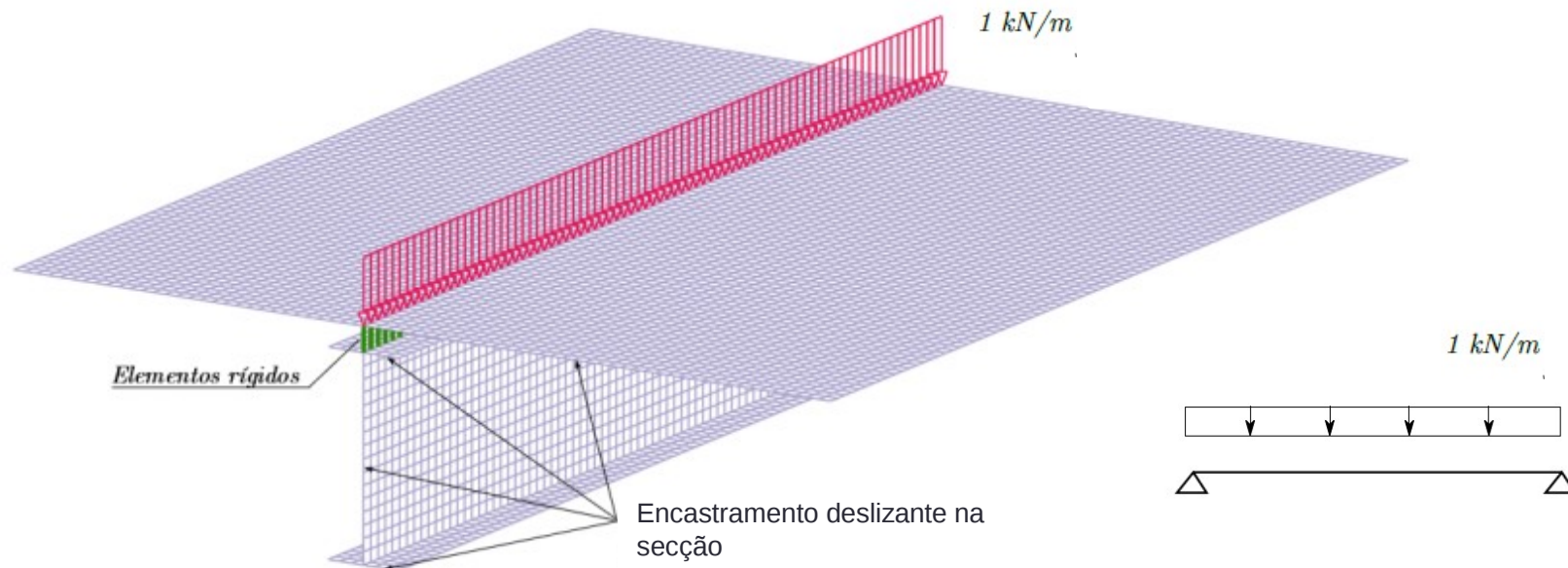
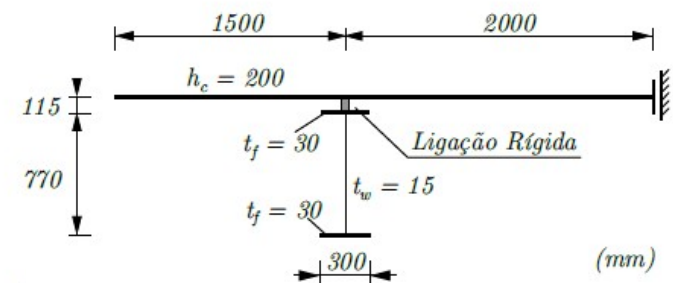
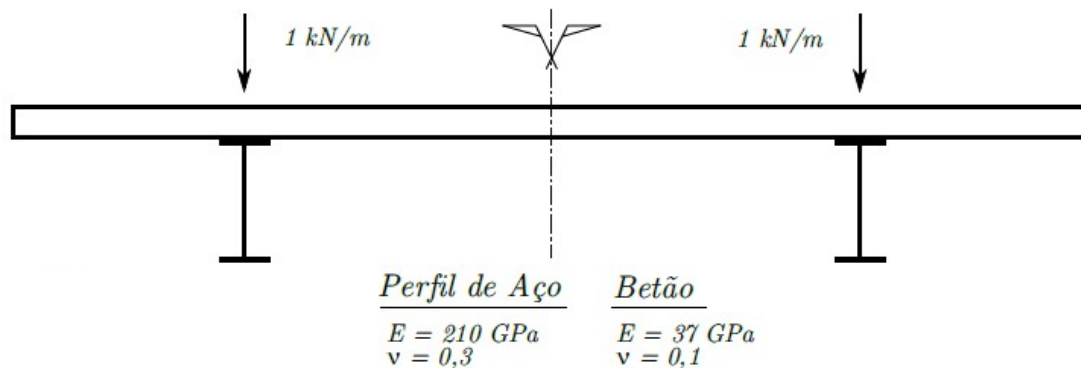


# Implementação MATLAB

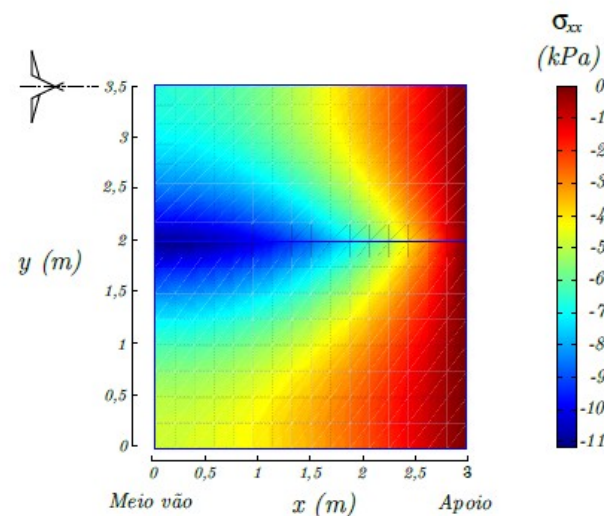
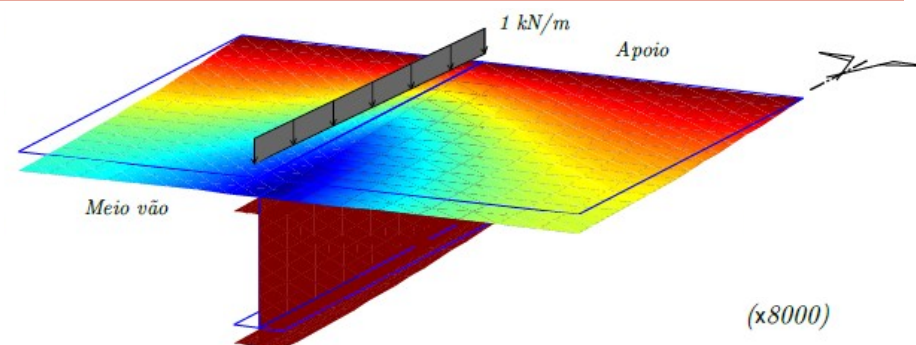
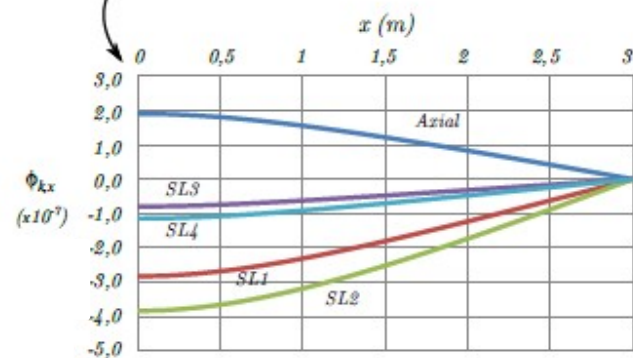
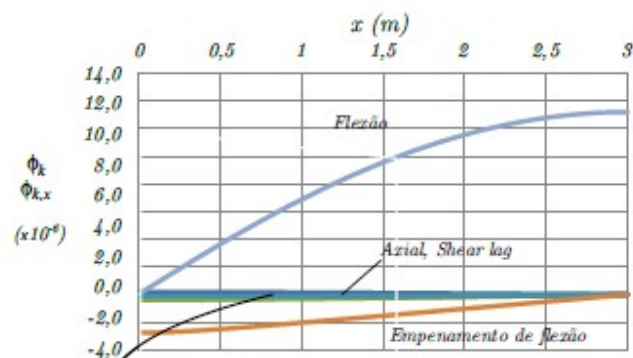
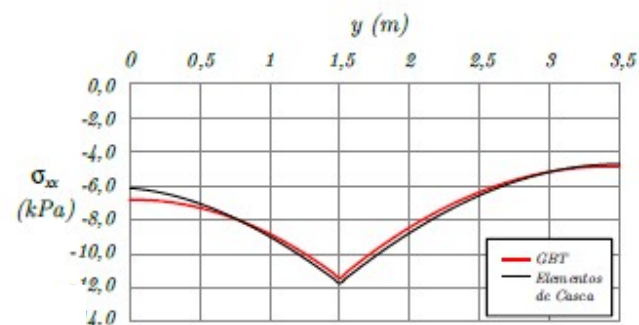
## Visualizador de resultados



# Exemplo 1

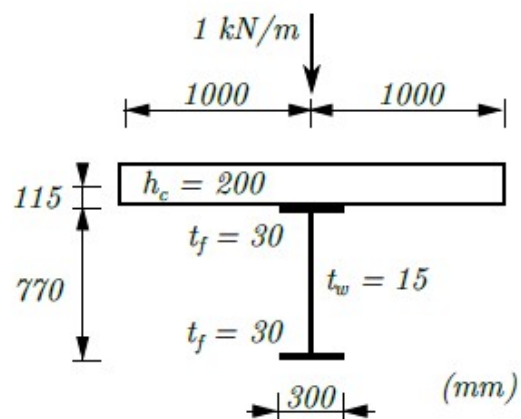


# Exemplo 1



	GBT	ADINA	Diferença relativa
Deslocamento	$1,32 \times 10^{-5}$	$1,31 \times 10^{-5}$	0,47 %

# Exemplo 2



Perfil de Aço

$$E = 210 \text{ GPa}$$

$$\nu = 0,3$$

Betão

$$E = 37 \text{ GPa}$$

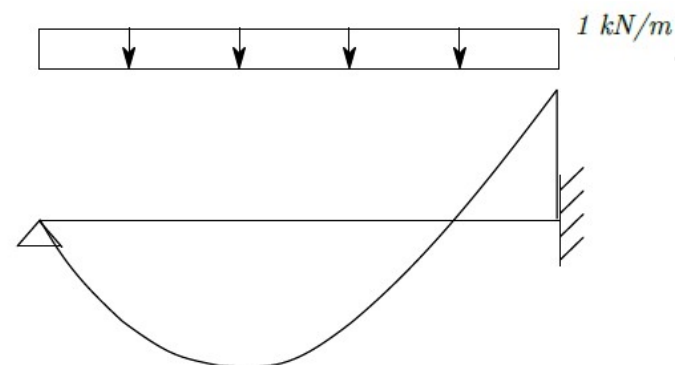
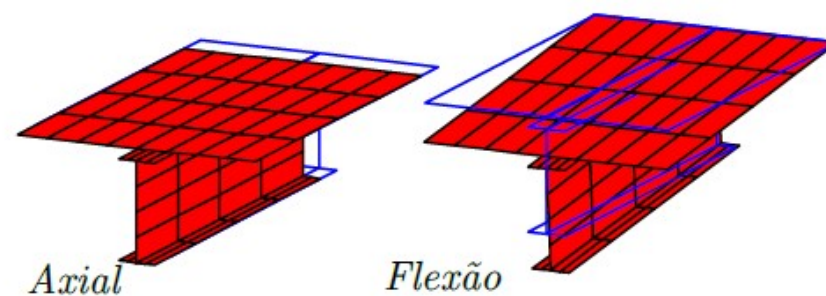
$$\nu = 0,2$$

Armadura

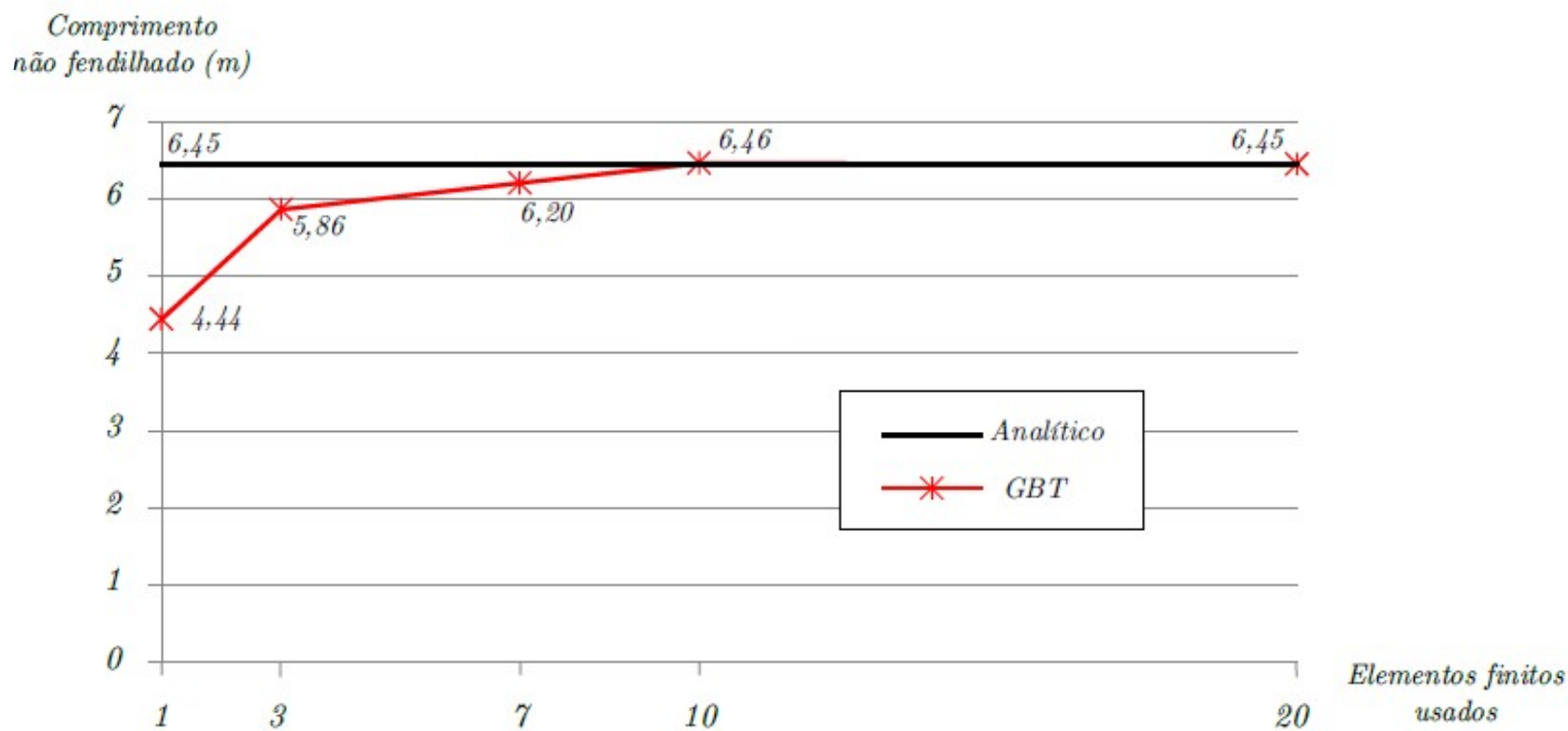
$$A = 15,71 \text{ cm}^2/\text{m}$$

$$E = 210 \text{ GPa}$$

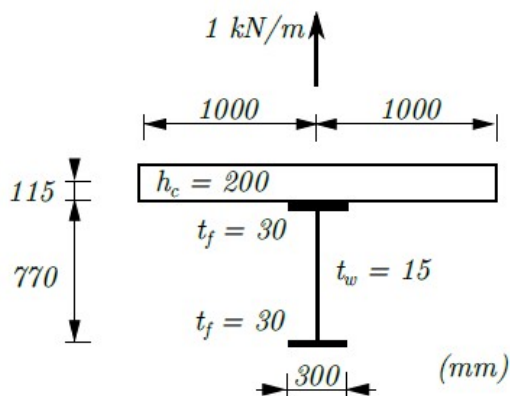
$$\nu = 0,3$$



# Exemplo 2



# Exemplo 3



Perfil de Aço

$$E = 210 \text{ GPa}$$

$$\nu = 0,3$$

Betão

$$E = 37 \text{ GPa}$$

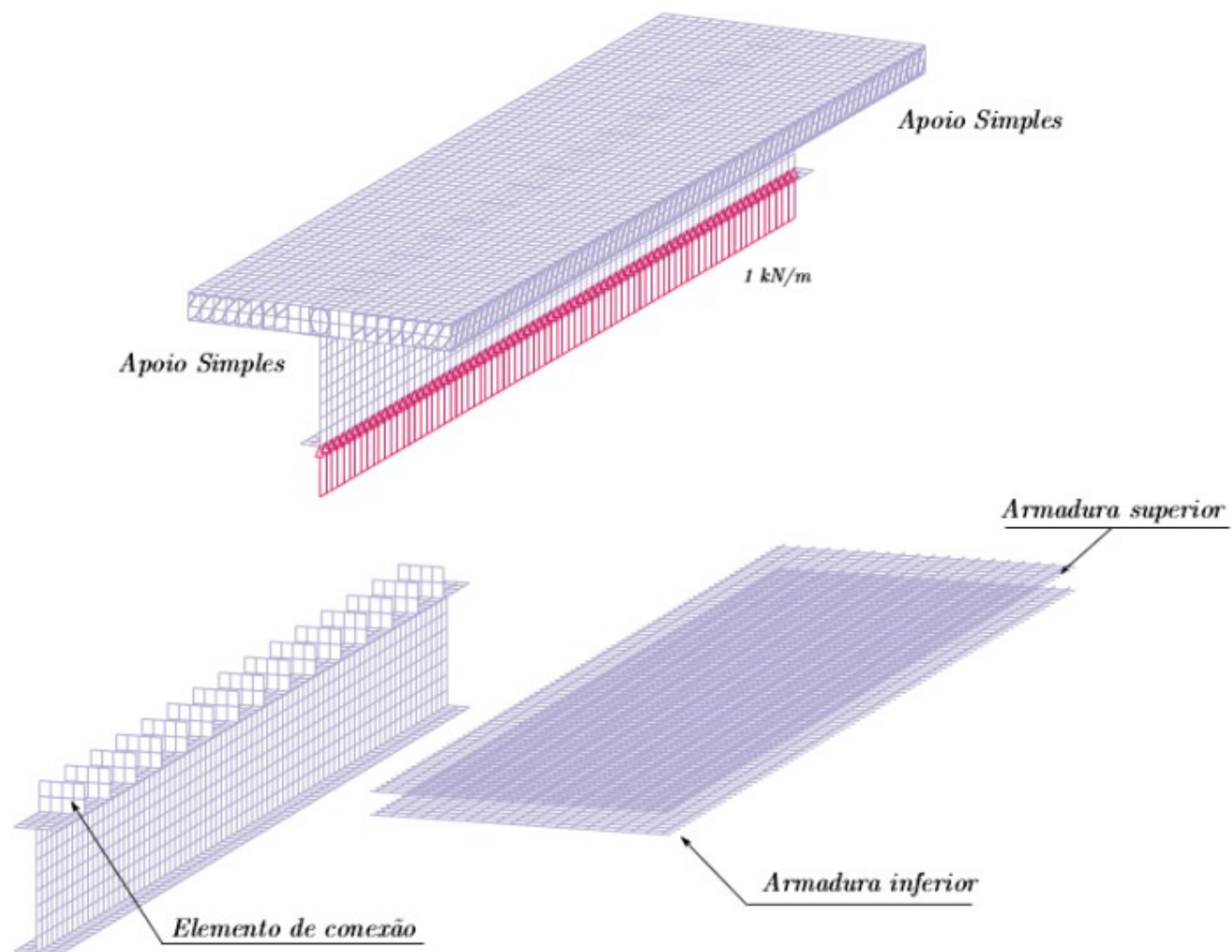
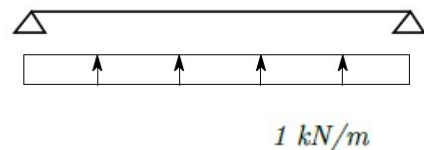
$$\nu = 0,2$$

Armadura

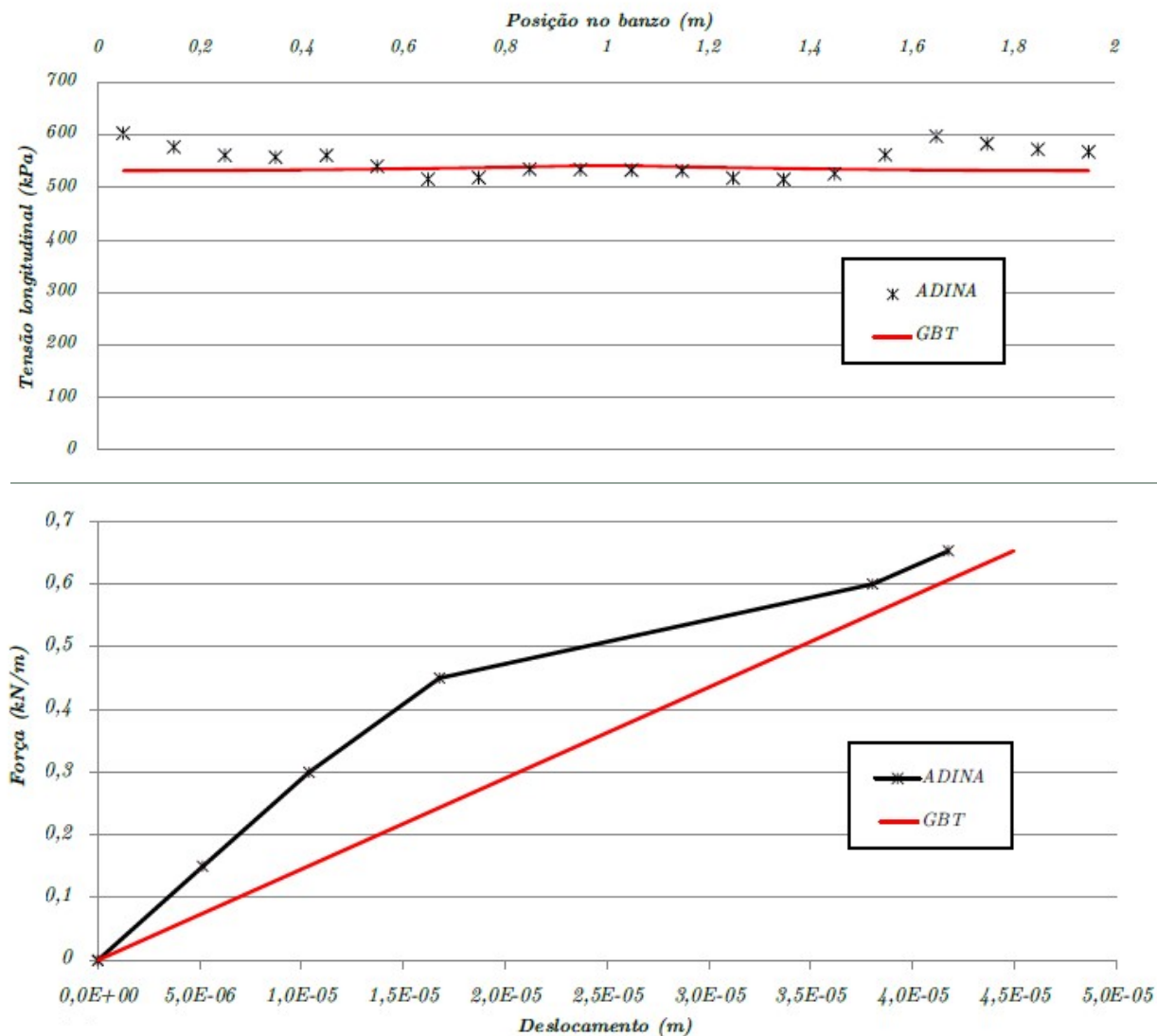
$$A = 15,71 \text{ cm}^2/\text{m}$$

$$E = 200 \text{ GPa}$$

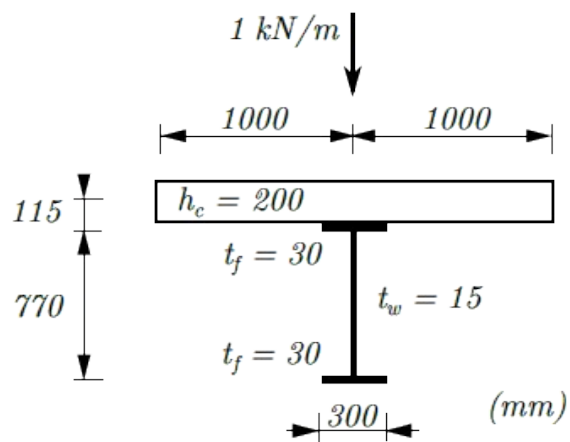
$$\nu = 0,3$$



# Exemplo 3



# Exemplo 4



## Perfil de Aço

$$E = 210 \text{ GPa}$$

$$\nu = 0,3$$

## Betão

$$E = 37 \text{ GPa}$$

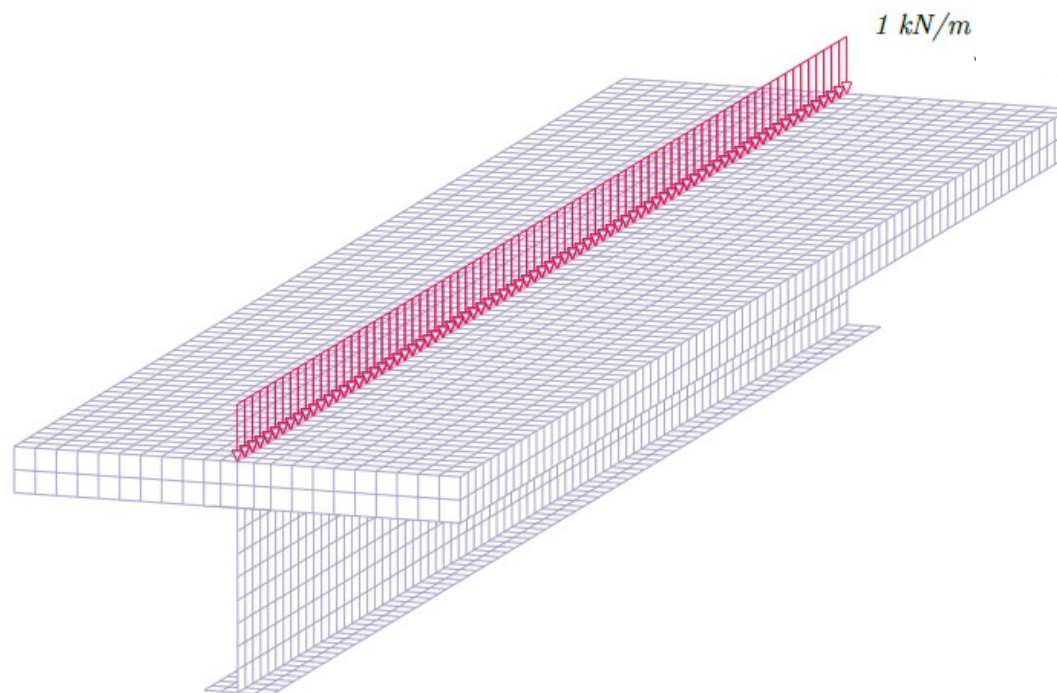
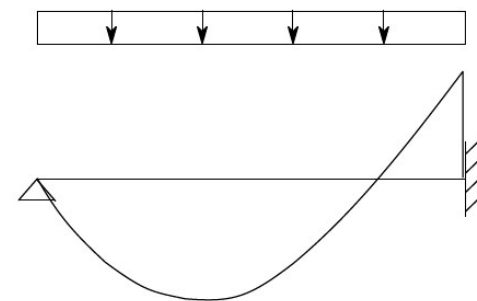
$$\nu = 0,2$$

## Armadura

$$A = 15,71 \text{ cm}^2/\text{m}$$

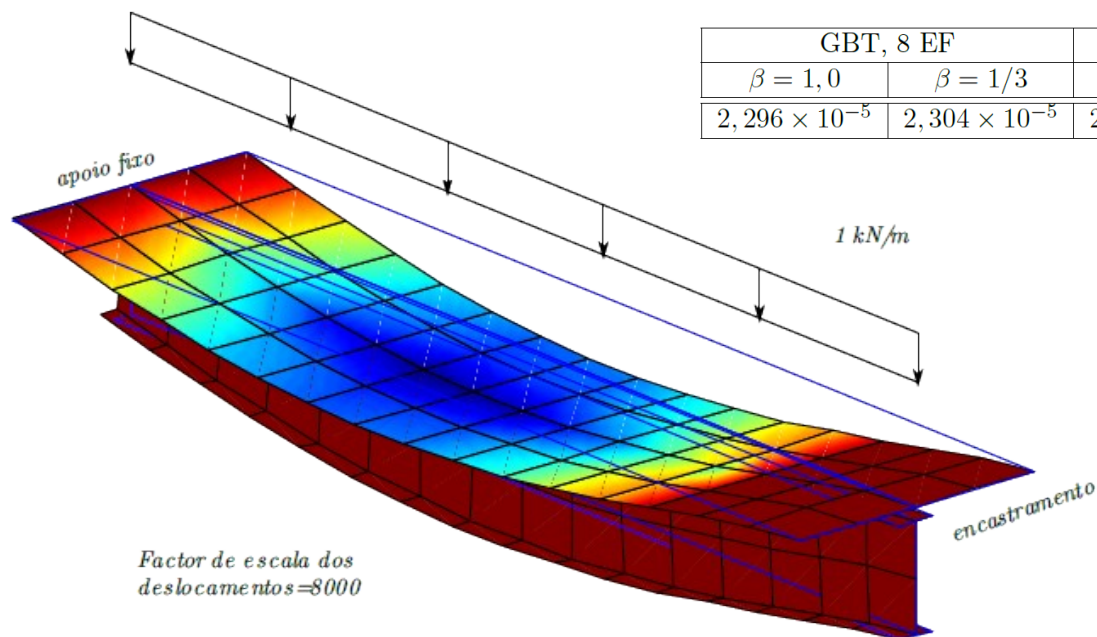
$$E = 200 \text{ GPa}$$

$$\nu = 0,3$$

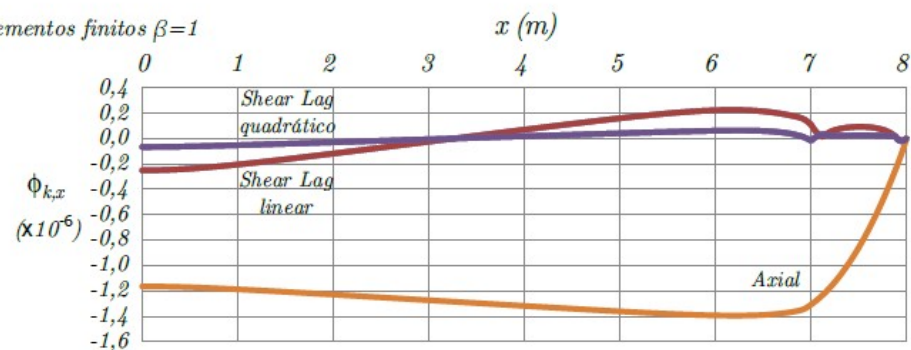
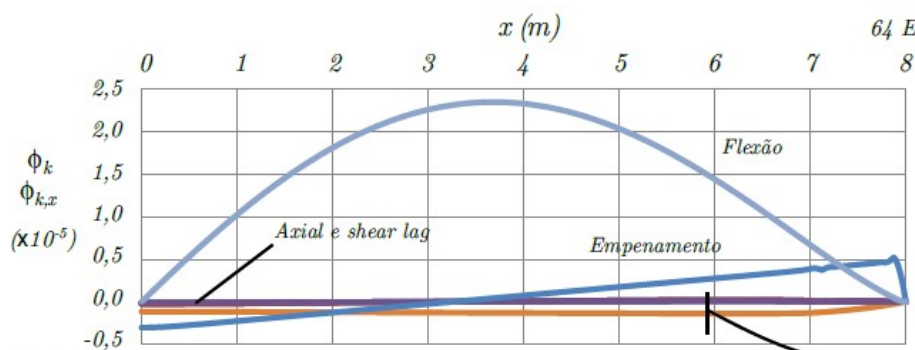




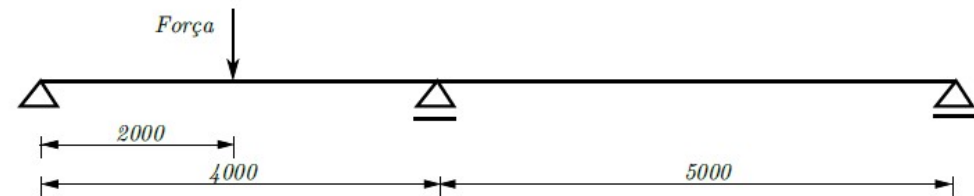
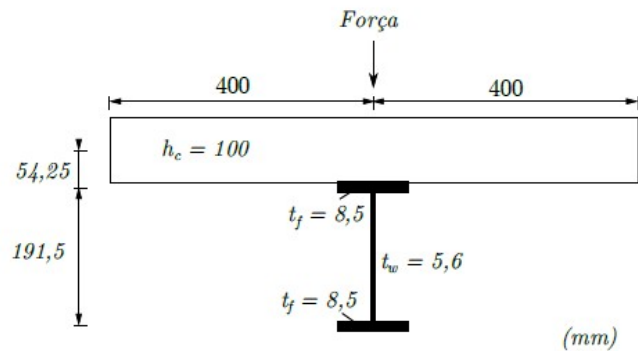
# Exemplo 4



GBT, 8 EF		GBT, 64 EF		ADINA
$\beta = 1,0$	$\beta = 1/3$	$\beta = 1,0$	$\beta = 1/3$	" $\beta$ " = 0.5
$2,296 \times 10^{-5}$	$2,304 \times 10^{-5}$	$2,331 \times 10^{-5}$	$2,332 \times 10^{-5}$	$2,183 \times 10^{-5}$



# Exemplo 5 – CTB1



### Perfil de aço

IPE200  
 $f_y = 285 \text{ MPa}$   
 $E = 200 \text{ GPa}$   
 $\nu = 0,3$

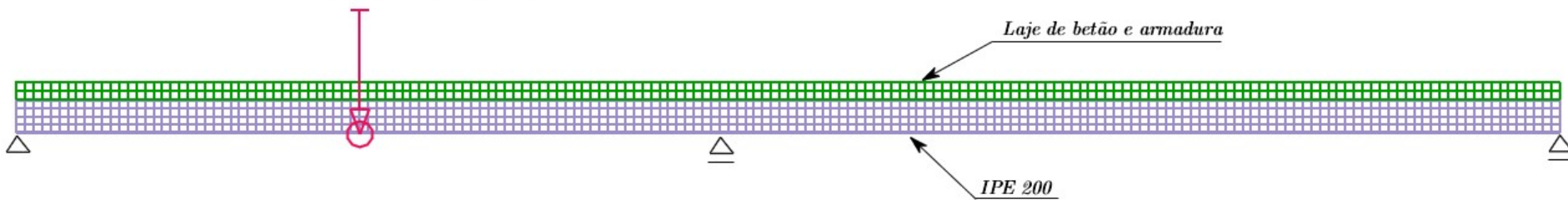
### Armadura

$f_y = 430 \text{ MPa}$   
 $E = 200 \text{ GPa}$   
 $\nu = 0,3$   
 $A_{sup} = 8,0 \text{ cm}^2$   
 $A_{inf} = 3,16 \text{ cm}^2$

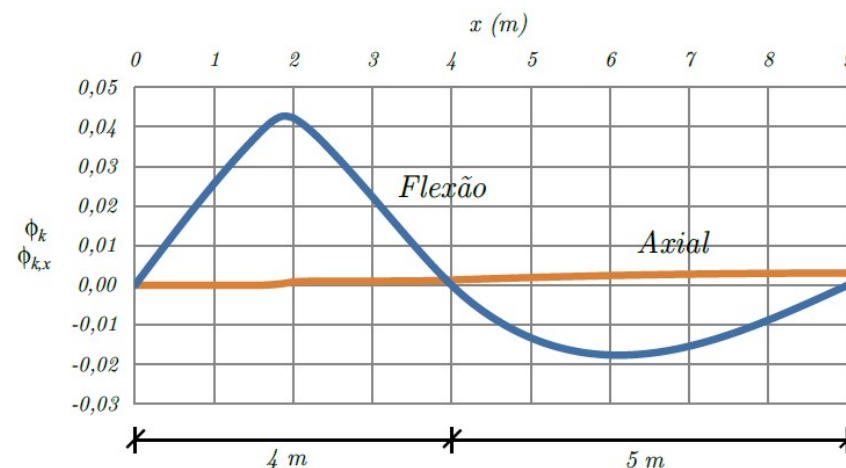
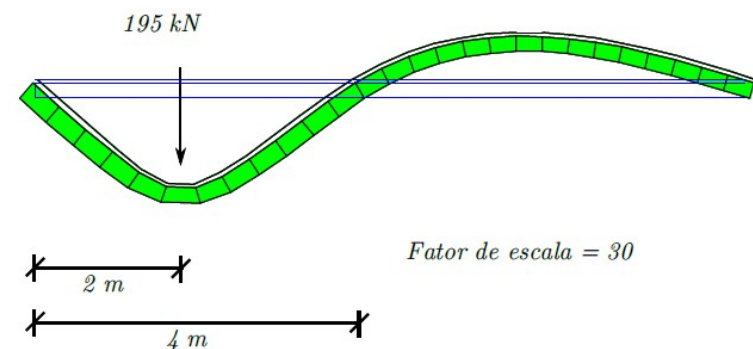
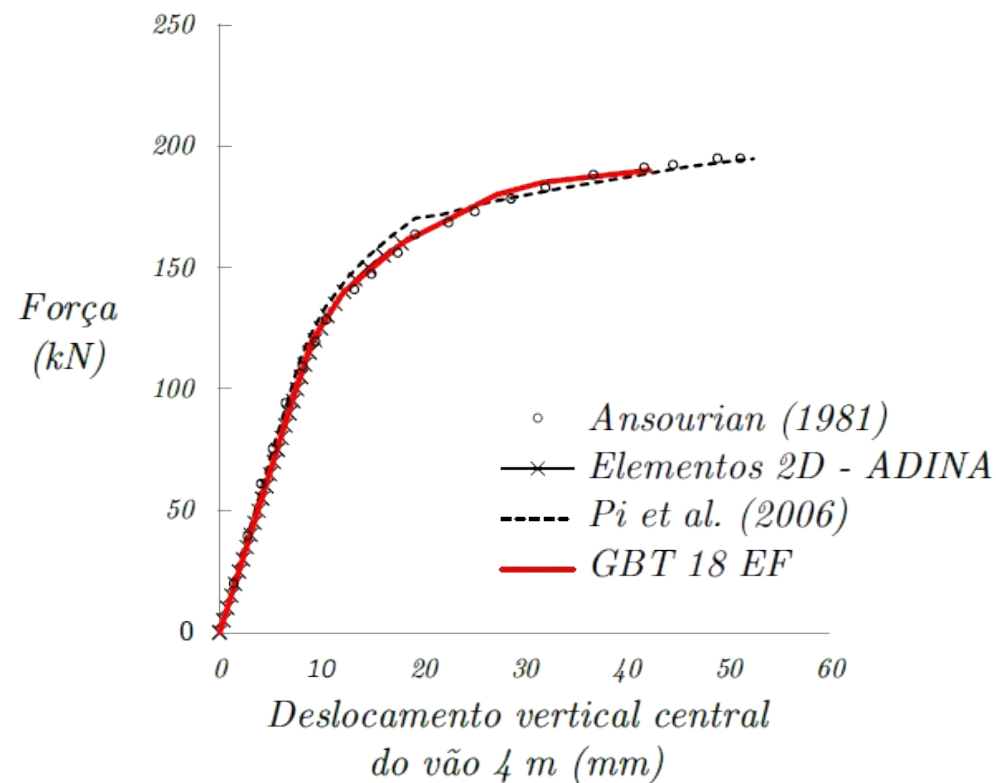
### Betão

$\sigma_c = 25,5 \text{ MPa}$   
 $\epsilon_c = 0,002$   
 $E_c = 24,1 \text{ GPa}$   
 $\nu = 0,2$

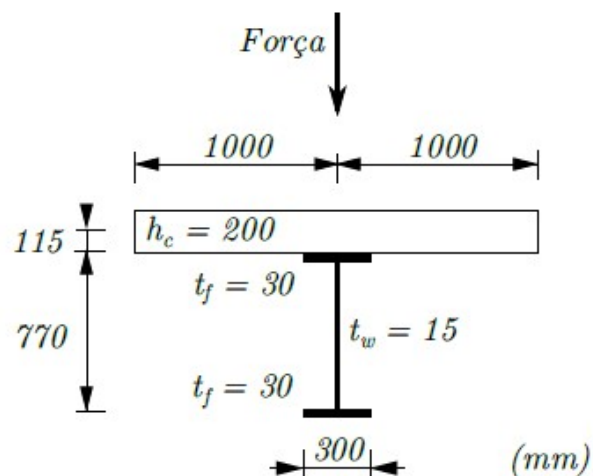
### *Deslocamento aplicado*



# Exemplo 5 – CTB1



# Exemplo 6



## Perfil de Aço

$$f_y = 235 \text{ MPa}$$

$$E = 210 \text{ GPa}$$

$$\nu = 0,3$$

## Armadura

$$f_y = 500 \text{ MPa}$$

$$E = 200 \text{ GPa}$$

$$\nu = 0,3$$

$$A = 22,62 \text{ cm}^2/\text{m}$$

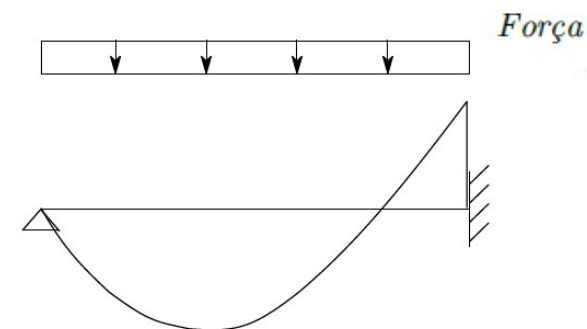
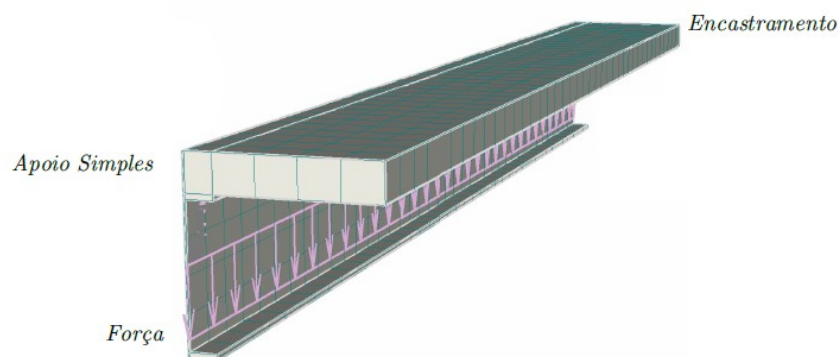
## Betão

$$\sigma_c = 50,15 \text{ MPa}$$

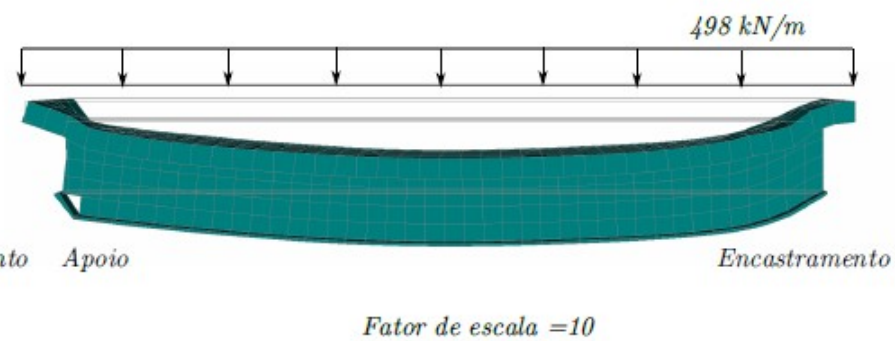
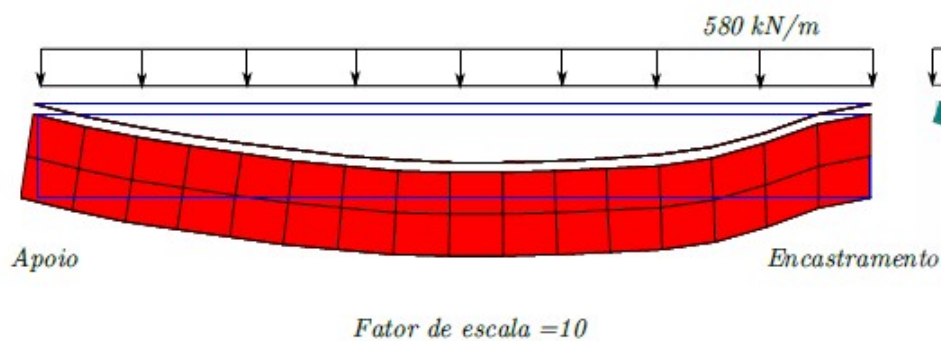
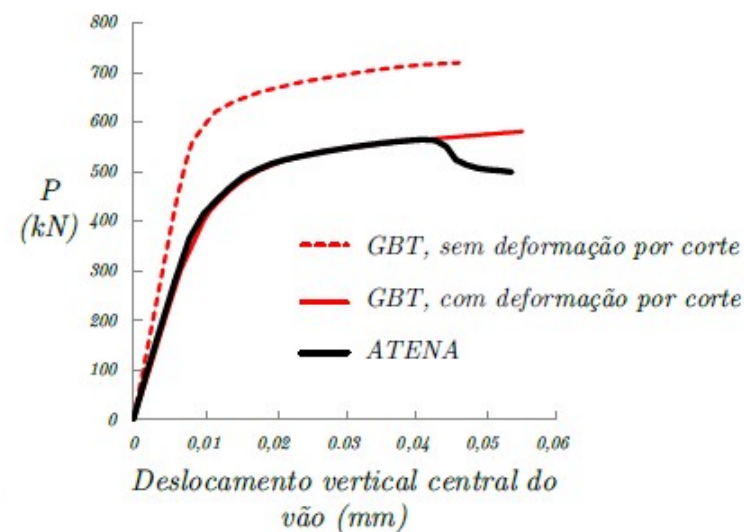
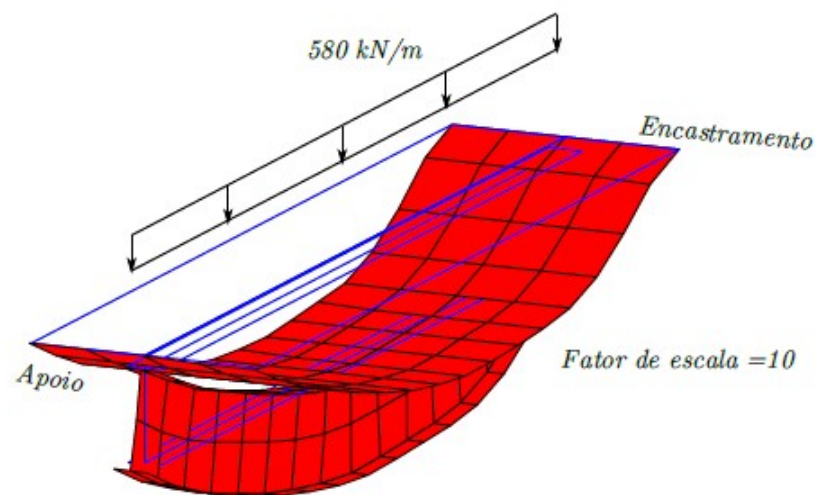
$$\epsilon_c = 0,00256$$

$$E_c = 39,06 \text{ GPa}$$

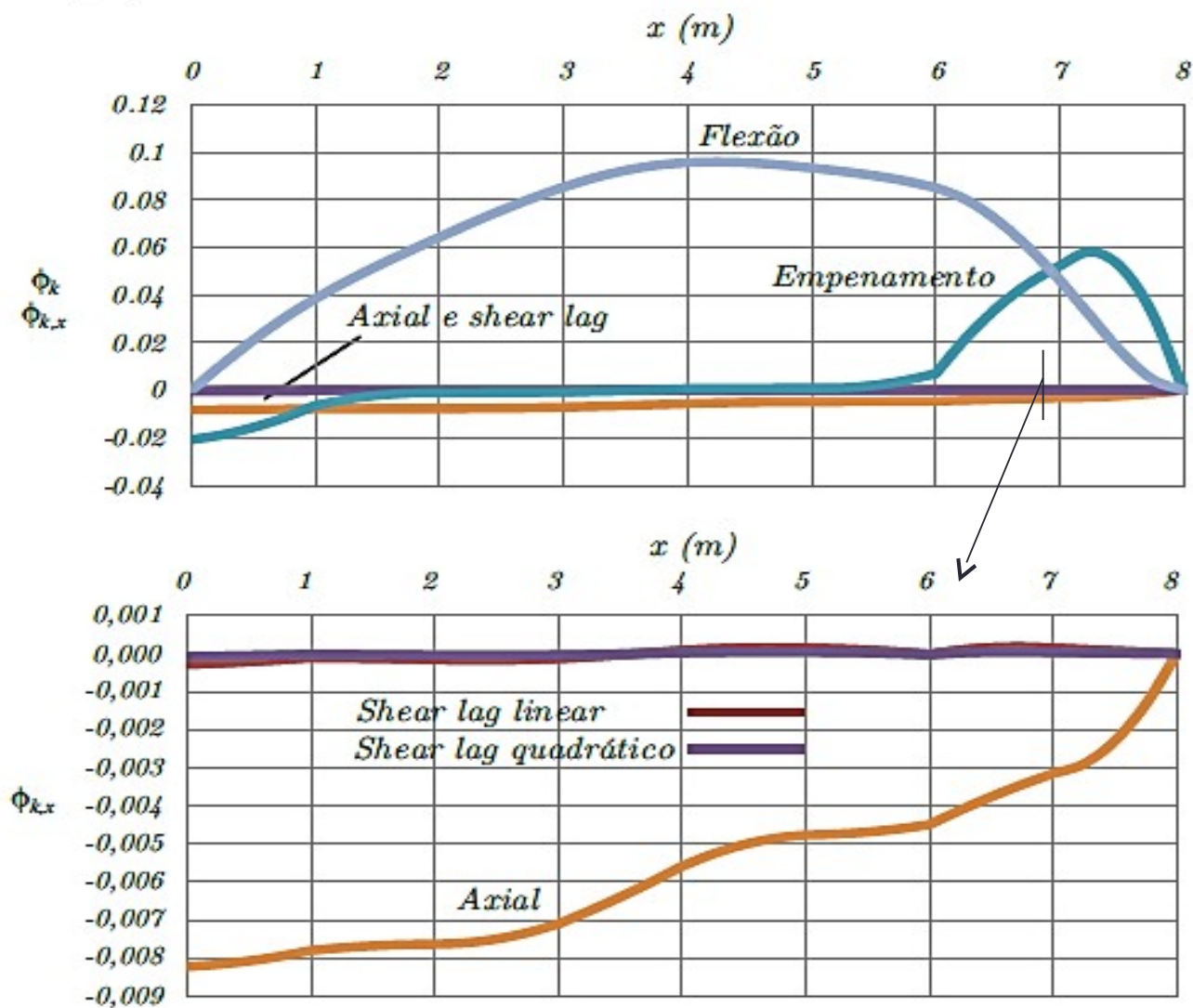
$$\nu = 0,2$$



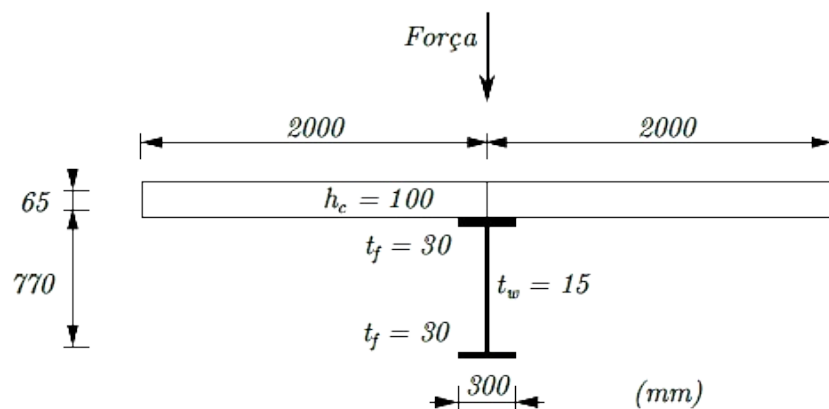
# Exemplo 6



# Exemplo 6



# Exemplo 7



## Perfil de Aço

$$f_y = 235 \text{ MPa}$$

$$E = 210 \text{ GPa}$$

$$\nu = 0,3$$

## Armadura

$$f_y = 500 \text{ MPa}$$

$$E = 200 \text{ GPa}$$

$$\nu = 0,3$$

$$A = 22,62 \text{ cm}^2/\text{m}$$

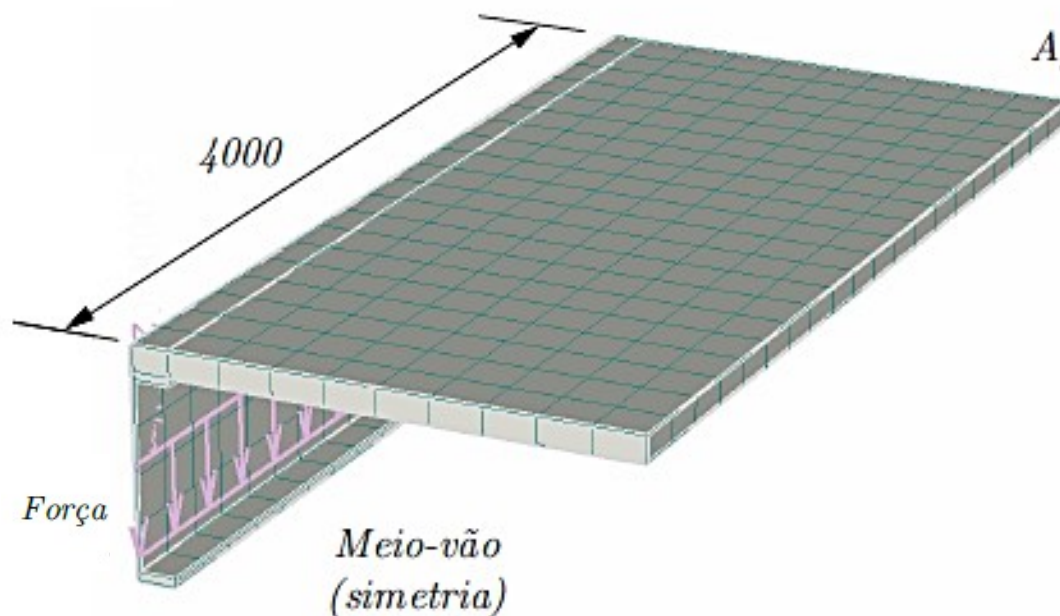
## Betão

$$\sigma_c = 50,15 \text{ MPa}$$

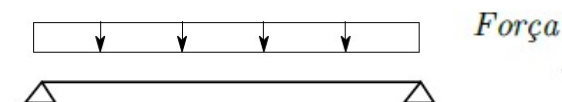
$$\epsilon_c = 0,00256$$

$$E_c = 39,06 \text{ GPa}$$

$$\nu = 0,2$$



*Apoio Simples*

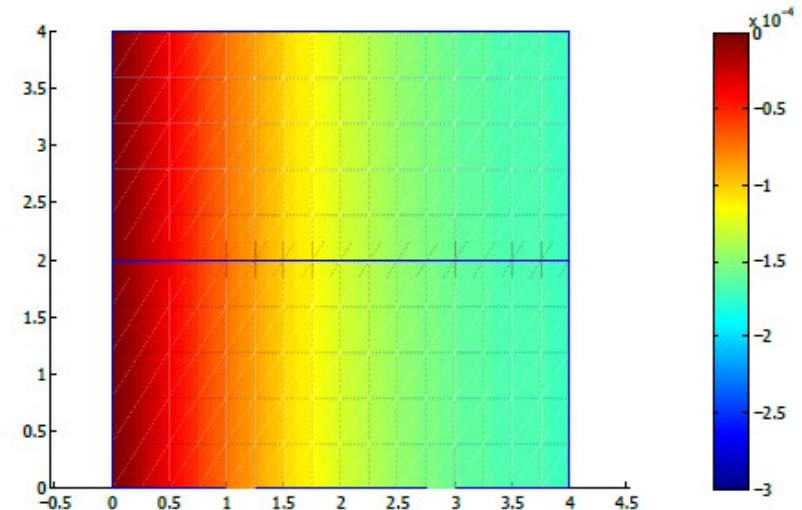
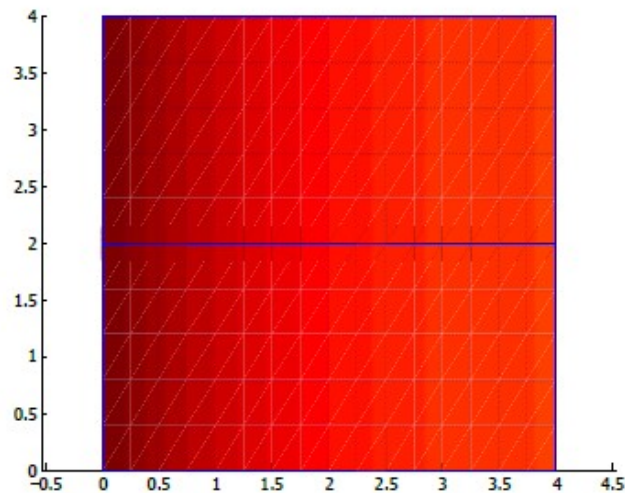


# Exemplo 7

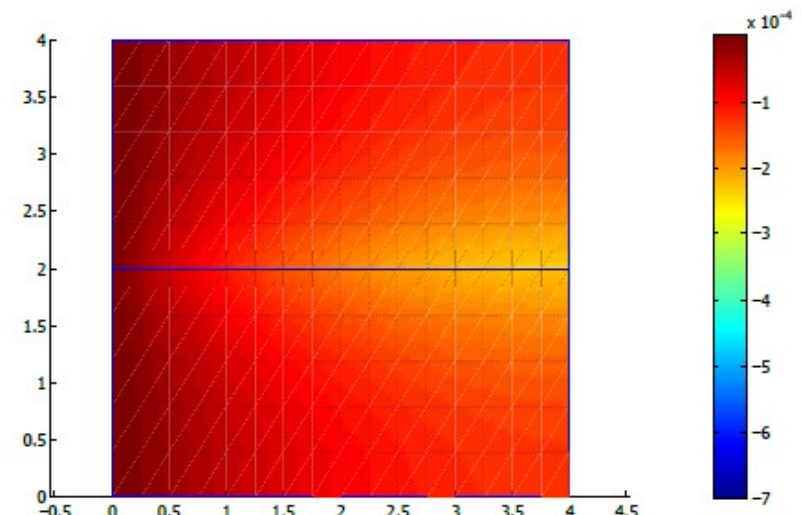
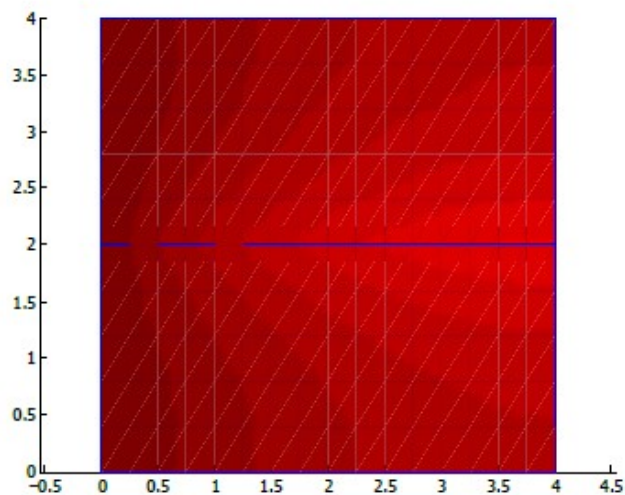
100 kN/m

300 kN/m

*Sem modos  
de shear lag*



*Com modos  
de shear lag*





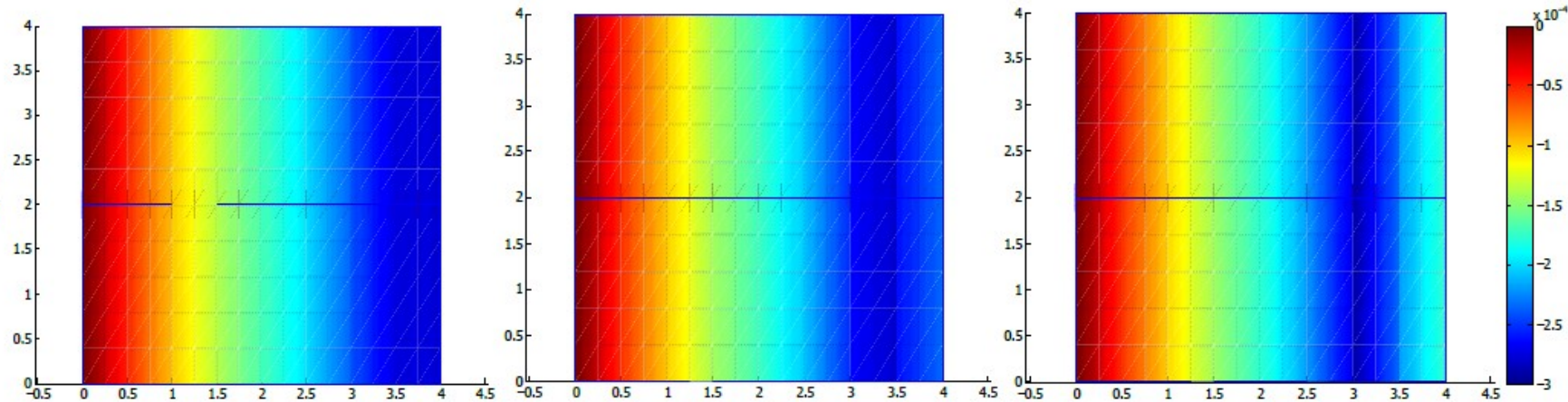
# Exemplo 7

400 kN/m

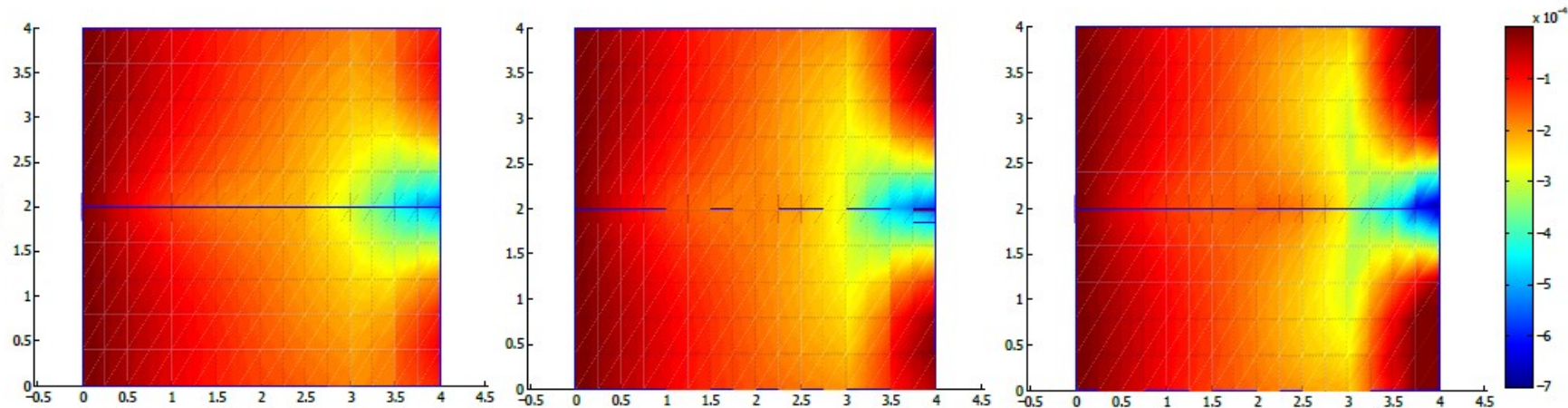
410 kN/m

420 kN/m

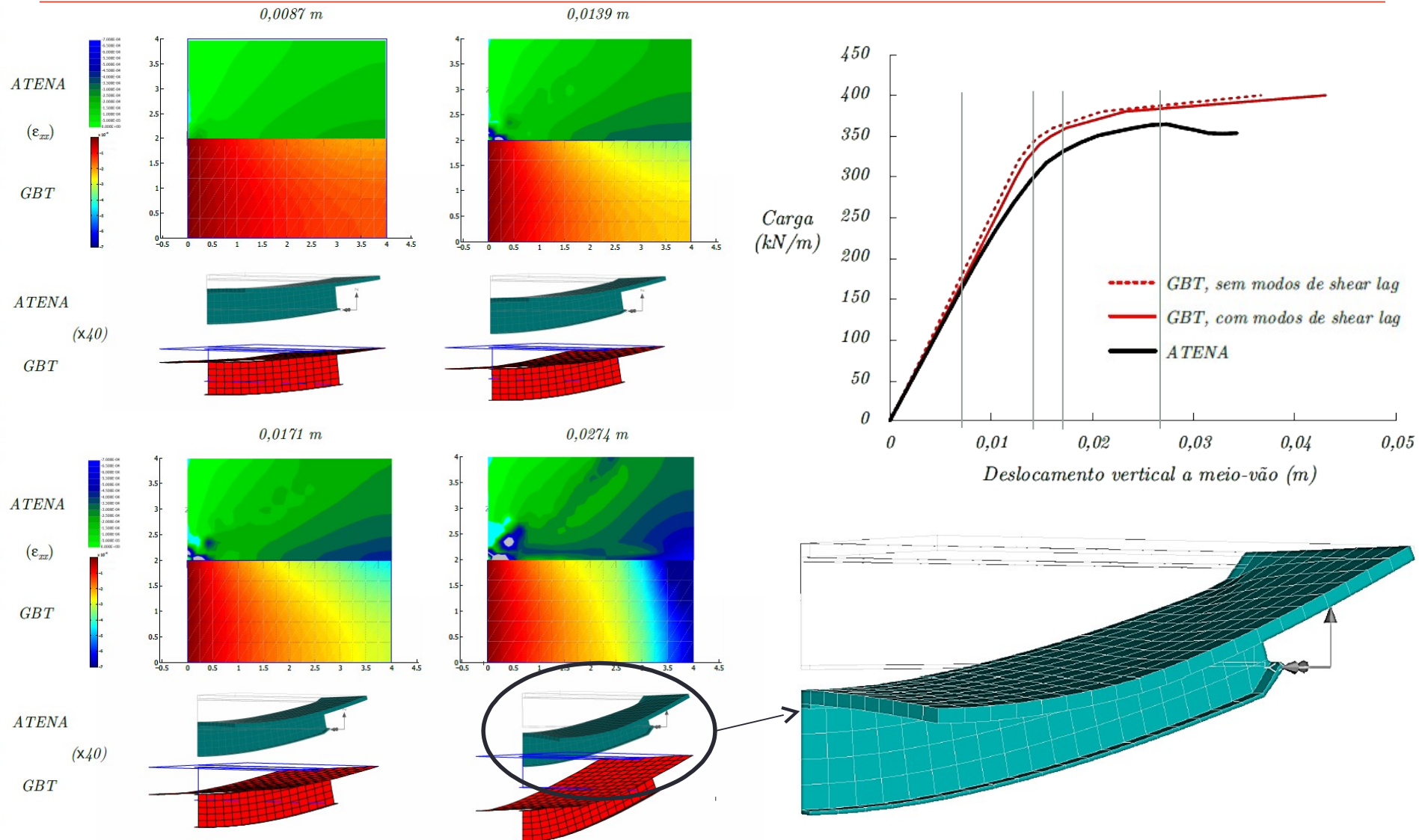
*Sem modos  
de shear lag*



*Com modos  
de shear lag*



# Exemplo 7



# Conclusões

---

- Elemento finito não-linear:
  - *Shear lag*;
  - Deformação por esforço transversal da alma do perfil de aço, fendilhação e comportamento não-linear à compressão do betão;
  - Plastificação do perfil e da armadura
- Eficácia computacional;
- Significado estrutural;
- Resultados muito semelhantes entre GBT e elementos convencionais.

# Desenvolvimentos Futuros

---

- Ao nível do programa:
  - Representação de grandezas plásticas;
  - Método Comprimento de Arco.
- Ao nível do elemento finito:
  - Efeitos diferidos no betão;
  - Efeitos geométricamente não-lineares;
  - Conexão de corte;
  - Flexão transversal da laje.